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## THE EFFECT OF A RANDOM NOISE BACKGROUND UPON THE DETECTION OF A RANDOM SIGNAL<sup>1</sup>

By H. S. HEAPS<sup>2</sup>

### ABSTRACT

A noise distributed in phase and power according to a Rayleigh law is studied in terms of its effect upon the detectability of a signal of similar phase and amplitude distribution. An expression is derived for the probability distribution of the ratio of the power of the signal plus noise to that of the noise in the absence of the signal. The corresponding result is given for the ratio of the averages over several observations. Also derived is the probability distribution of the fractional change in noise plus signal power due to a given fractional change in signal power.

### 1. INTRODUCTION

When a signal is received against a background of 'noise' due to reverberation, scattering of the original signal, or to the presence of an unwanted signal, the observable quantity is the sum of the signal and the noise. Fluctuation of this sum is dependent upon the fluctuation of the signal and of the noise background, both of which at any place and time are characterized by their amplitude, phase, and frequency distribution. Some statistical properties of the sum of a sinusoidal signal and a normally distributed noise background have been obtained by Rice (6, 7).

Throughout the present paper the signal and the noise at any instant are represented in the complex form  $p^{1/2}\exp(i\theta)$ , where  $p$  is real and is here termed the power at that instant. Recognition of the signal is dependent upon the sensitivity of the receiver to differences in amplitude and frequency. The ease of detection of a pulse may depend upon the difference of the averages over the pulse length of the power of the signal plus noise and of the noise alone. If the noise and signal have the same frequency, or if the receiver is sensitive to a single frequency, then recognition of the signal depends upon the ratio of the power of the signal plus noise background to that of the noise in the absence of the signal; in the case of a pulse of short duration during which the phenomena producing the signal and the noise do not change, the averaging over a pulse length consists of the replacement of the time varying components by a constant so that it is sufficient to consider a single instant during reception of the pulse. The question as to what change in power received is a sufficient indication of the presence of a signal is discussed by Lawson and Uhlenbeck (1,

<sup>1</sup>Manuscript received August 23, 1954.

Contribution from the Naval Research Establishment, Dartmouth, N.S.

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Ch. 6) in terms of the ratio of the change in power to the standard deviation of the noise. The present paper relates a fractional change in the mean signal power to the resulting fractional change in the received signal plus noise background.

Consider an instantaneous value of the noise. At a succeeding instant separated from the first by a very small time interval it appears reasonable to suppose that the noise power remains unchanged but that the phase is increased by a periodic time factor. Between two instants separated by a sufficiently large time interval, however, it seems more reasonable to assume that the two values of the noise are independent and may be related only statistically. The same remarks apply to the values of the signal considered at two separate instants. Thus a knowledge of the noise and signal at one particular instant makes it possible to infer their values within a sufficiently small time range but not at times sufficiently remote. For a pulsed signal of length sufficiently small so that the propagating medium does not vary throughout the duration of the pulse, the small time interval described above may be much smaller than the pulse length owing to the different paths traversed by the sound received at any instant. In such a case the received signal may be regarded as a superposition of individual pulses of duration less than the small time intervals within which inference from an instantaneous value is possible. Under similar assumptions Mintzer (3, 4, 5) has considered the propagation of a series of pulses and their statistical variation due to variations in the refractive index of the medium.

In the present paper, since it is supposed that the fluctuation of both the noise and the signal is due to the superposition of several effects, it will be assumed that at any instant the variation of power is according to a Rayleigh distribution and that all phase values are equally likely. By virtue of the equation [2.9] of Wallace (8) it follows that the variation of noise received from small random scatterers is according to a Rayleigh law provided the number of scatterers is large (see also Marshall and Hitschfeld (2)).

The probability that, at a given instant, the noise plus signal power is a given multiple of the noise power is obtained in Section 2. When the signal has the same frequency as the noise the result may be used to compare the time averages over a sufficiently small duration. The averages over  $r$  instants contained in different pulses are compared in Section 3, the result also applying in the case of a common signal and noise frequency to averages over  $r$  short intervals. Comparison of the interval averages is made with the case in which the noise and signal frequencies are different.

If, during reception of a pulse, the signal power changes by a factor  $f^2$  then the signal plus noise received changes by a factor  $h^2$  whose probability distribution is dependent upon  $f$  and also upon the relative magnitudes of the signal and the noise power. The probability of a given value of  $h$  is obtained in Section 4.

## 2. ANALYSIS AT A SINGLE INSTANT

For a signal of phase and power distributed according to a Rayleigh law, the probability, at any instant, that the signal power  $p$  lies between  $p$  and  $p+dp$  with phase between  $\theta$  and  $\theta+d\theta$  is



$$[1] \quad (1/2\pi\bar{p})\exp(-p/\bar{p})dpd\theta$$

in terms of the mean power  $\bar{p}$ . The corresponding expression for a background noise of similar Rayleigh distribution is

$$[2] \quad (1/2\pi\bar{p}_n)\exp(-p_n/\bar{p}_n)dp_nd\theta.$$

At an instant when the noise power lies between  $p_n$  and  $p_n+dp_n$  the probability that the signal plus noise has power between  $q$  and  $q+dq$  and phase between  $\phi$  and  $\phi+d\phi$  in excess of the noise is

$$[3] \quad (1/2\pi\bar{p})\exp[(-p_n-q+2p_n^{1/2}q^{1/2}\cos\phi)/\bar{p}]dq d\phi.$$

Accordingly, the probability that the noise power lies between  $p_n$  and  $p_n+dp_n$  while the power of the signal plus noise has power between  $q$  and  $q+dq$  is

$$[4] \quad (1/2\pi\bar{p}\bar{p}_n)\exp[-p_n/\bar{p}_n-(p_n+q)/\bar{p}]dp_ndq \int_0^{2\pi} \exp[2p_n^{1/2}q^{1/2}\cos\phi/\bar{p}]d\phi \\ = (1/\bar{p}\bar{p}_n)\exp[-p_n/\bar{p}_n-(p_n+q)/\bar{p}].I_0(2p_n^{1/2}q^{1/2}/\bar{p})dp_ndq,$$

where  $I_0$  denotes the modified Bessel function of zero order (9, p. 181). Putting  $q = kp_n$  and  $dq = p_n dk$ , the probability that the ratio of the signal plus noise power to the noise power lies between  $k$  and  $k+dk$  is  $P(k)dk$ , where

$$[5] \quad P(k) = (1/\bar{p}\bar{p}_n) \int_0^\infty \exp[-\{1/\bar{p}_n+(1+k)/\bar{p}\}p_n]I_0(2k^{1/2}p_n/\bar{p})p_ndp_n.$$

The last integral is of the form

$$[6] \quad \int_0^\infty \exp(-ap_n)J_0(ibp_n)p_ndp_n,$$

which has the value  $a/(a^2-b^2)^{3/2}$  valid for  $a > b$  (9, p. 386). Substitution of this expression, with the appropriate values of  $a$  and  $b$ , into the equation for  $P(k)$  leads to

$$[7] \quad P(k) = \bar{p}\bar{p}_n(\bar{p}_nk+\bar{p}+\bar{p}_n) [(\bar{p}_nk+\bar{p}-\bar{p}_n)^2+4\bar{p}\bar{p}_n]^{-3/2}.$$

The validity condition  $a > b$  is equivalent to the condition  $(\bar{p}_nk+\bar{p}-\bar{p}_n)^2+4\bar{p}\bar{p}_n > 0$  which is certainly fulfilled. Denoting  $\bar{p}/\bar{p}_n$  by  $c$ , the expression [7] takes the form

$$[8] \quad P(k) = c(k+c+1)[(k+c-1)^2+4c]^{-3/2},$$

which for values of  $c$  less than 2 has a maximum at  $k = \frac{1}{2}(9+8c)^{1/2}-c-\frac{1}{2}$ , and which for values of  $c$  greater than 2 is a monotonically decreasing function of  $k$ .

For values of  $c \ll 1$  the expression [8] may be written as

$$[9] \quad P(k) = c(k+1) |k-1|^{-3},$$

where  $|k-1|$  denotes the numerical value of  $k-1$ . When  $c \gg 1$  the corresponding form of equation [8] is

$$[10] \quad P(k) = c(k+c)^{-2}.$$

The function [10], together with the function [8] when  $c = 0.1, 1$ , and  $2$ , is plotted against  $k$  as a broken curve in Figs. 1-4. In the graph for  $c \gg 1$  the horizontal scale is in terms of units of  $c$ .

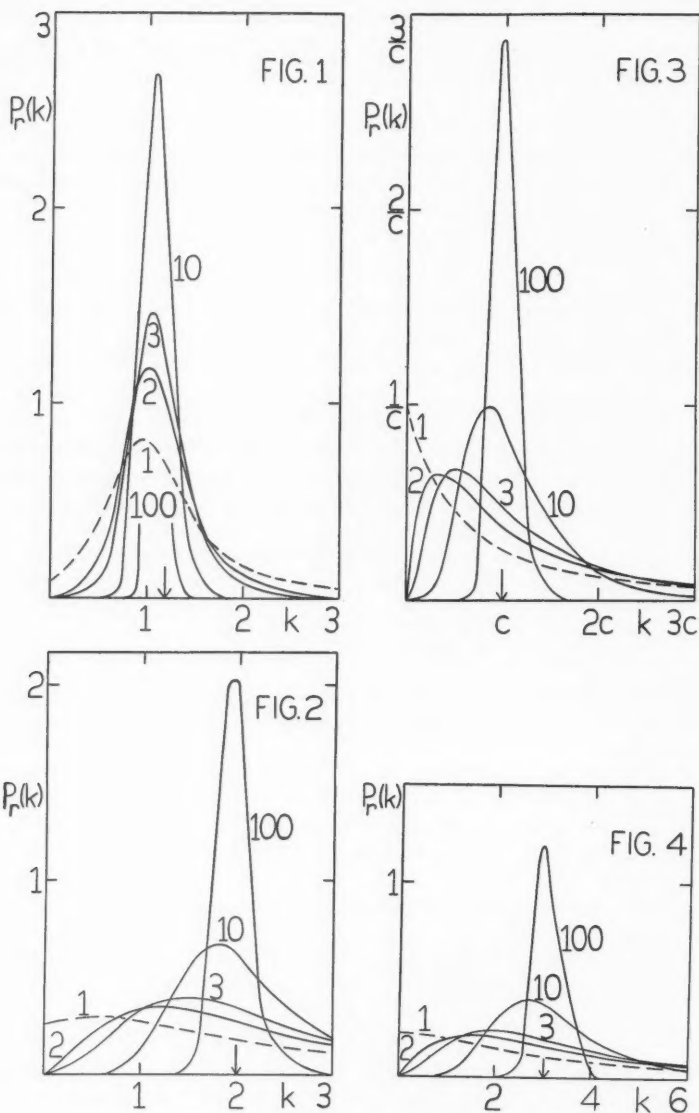


FIG. 1. Probability distribution of  $P_r(k)$ , the probability that the signal plus noise power averaged over  $r$  random samples is equal to  $k$  times the averaged noise power (Equation [16]). On each curve the value of  $r$  is indicated.  $c$ , the mean signal power/mean noise power, is 0.1.

FIG. 2. Probability distribution of  $P_r(k)$  when  $c = 1$ .

FIG. 3. Probability distribution of  $P_r(k)$  when  $c \gg 1$ .

FIG. 4. Probability distribution of  $P_r(k)$  when  $c = 2$ .

The quantity  $P(k)dk$  is the probability that at any instant the ratio of the signal plus noise power to the power of the background noise lies within the limits  $k$  and  $k+dk$ . The probability that this ratio is greater than  $k$  is given by

$$[11] \quad Q(k) = \int_0^\infty P(k)dk = \frac{1}{2} + \frac{1}{2}(c-k+1)[(k+c-1)^2+4c]^{-1/2}.$$

The value of  $\frac{1}{2}$  is attained by  $Q(k)$  when  $k = c+1$ , in agreement with the fact that the median value of  $q$  is  $\bar{p} + \bar{p}_n$ ; such values of  $k$  occur below the arrows in Figs. 1-4.

### 3. RANDOM AVERAGES

If the signal plus noise is observed at  $r+1$  distinct instants separated by sufficiently large time intervals for there to be no correlation, other than statistical, between the signal values or the noise values from one chosen instant to another, then the signal plus noise power averaged over the  $r+1$  instants has a probability distribution whose form depends upon the value of  $r$ . Likewise the power of the noise alone averaged over the  $r+1$  instants has a probability distribution dependent upon  $r$ .

The probability that the sum of the noise powers lies between  $p_n$  and  $p_n+dp_n$ , while the sum of the signal plus noise powers lies between  $q$  and  $q+dq$ , may be derived analogously to the expression [4] and is

$$[12] \quad dp_n dq (\bar{p}\bar{p}_n)^{-r-1} \exp[-p_n/\bar{p}_n - (p_n+q)/\bar{p}] \int \dots \int I_0(2x_1^{1/2}y_1^{1/2}/\bar{p}) \\ \times I_0(2[x_2-x_1]^{1/2}[y_2-y_1]^{1/2}/\bar{p}) \dots I_0(2[p_n-x_r]^{1/2}[q-y_r]^{1/2}/\bar{p}) dx_1 dy_1 \dots dx_r dy_r,$$

where the integrations are over  $0 < x_1 < x_2 < \dots$  and  $0 < y_1 < y_2 < \dots$ . Repeated use of the identity

$$[13] \quad \int_0^t \int_0^s (xy)^{r/2} I_r(2x^{1/2}y^{1/2}/\bar{p}) I_0(2[s-x]^{1/2}[y-t]^{1/2}/\bar{p}) dx dy \\ = \bar{p}(r+1)^{-1} (st)^{1/2(r+1)} I_{r+1}(2s^{1/2}t^{1/2}/\bar{p}),$$

proved by applying the Faltung Theorem for the double Laplace transform to the functions  $(xy)^{r/2} I_r$  and  $I_0$ , reduces [12] to the form

$$[14] \quad dp_n dq (\bar{p}\bar{p}_n)^{-r-1} \exp[-p_n/\bar{p}_n - (p_n+q)/\bar{p}] (\bar{p}^r/r!) (p_n q)^{r/2} I_r(2p_n^{1/2}q^{1/2}/\bar{p}).$$

Putting  $q = k p_n$  and  $dq = p_n dk$ , the probability that the ratio of the average signal plus noise power to the average noise power lies between  $k$  and  $k+dk$  is given by  $P_{r+1}(k)dk$  where

$$P_{r+1}(k) = (\bar{p}\bar{p}_n)^{-r-1} (\bar{p}^r/r!) k^{r/2} \int_0^\infty \exp[-p_n/\bar{p}_n - p_n(1+k)/\bar{p}] p_n^{r+1} I_r(2k^{1/2}p_n/\bar{p}) dp_n \\ [15] \quad = (2r+1)!(r!)^{-2} (\bar{p}\bar{p}_n)^{r+1} k^r (\bar{p} + \bar{p}_n + k\bar{p}_n) [(\bar{p} + \bar{p}_n + k\bar{p}_n)^2 - 4k\bar{p}_n^2]^{-r-3/2}$$

as in Reference 9, p. 386. Denoting  $\bar{p}/\bar{p}_n$  by  $c$ , equation [15] becomes

$$[16] \quad P_{r+1}(k) = (2r+1)!(r!)^{-2} c^{r+1} k^r (k+c+1) [(k+c-1)^2+4c]^{-r-3/2},$$

which, for large values of  $r$  by virtue of Stirling's formula, may be written as

$$P_r(k) = (1/\pi) r^{1/2} 2^{2r-1} e^{-c} k^r (k+c+1) [(k+c-1)^2+4c]^{-r-1/2}$$

with a percentage error of less than  $10^4/r$ .

The graphs of  $P_r(k)$  for  $c = 0.1, 1, 2$ , and  $\gg 1$  are drawn in Figs. 1-4 for  $r$  values of 1, 2, 3, 10, and 100. The positions of the modal values of  $k$  when  $c = 0, 1, 2$ , and  $\gg 1$  are listed in Table I.

TABLE I  
MODAL VALUES OF  $k$  WITH RESPECT TO  $P_r(k)$

$r$	$c = 0$	$c = 1$	$c = 2$	$c \gg 1$
1	1	0.56	0	$c$
2	1	1.24	1.52	$0.33c$
3	1	1.45	1.93	$0.50c$
10	1	1.82	2.63	$0.82c$
100	1	1.98	2.96	$0.98c$

The probability that the ratio of the noise plus signal power averaged over  $r$  random samples is greater than  $k$  times the averaged noise power is given by

$$[17] \quad Q_r(k) = \int_k^{\infty} P_r(k) dk,$$

which becomes an increasingly complicated function as  $r$  increases. For  $r = 2$  the expression is

$$[18] \quad Q_2(k) = \frac{1}{2} + \frac{1}{2}(c-k+1) [(k+c-1)^2 + 4c]^{-3/2} [(k+2c-1)^2 + 3c(2-c)],$$

while for  $r = 3$  it is

$$[19] \quad Q_3(k) = \frac{1}{2} + \frac{1}{2}(c-k+1) [(k+c-1)^2 + 4c]^{-5/2} [u^4 + 2cu^3 + 2c(5+2c)u^2 - 4c^2(3c-5)u + 2c^2(3c^2 - 10c + 15)]$$

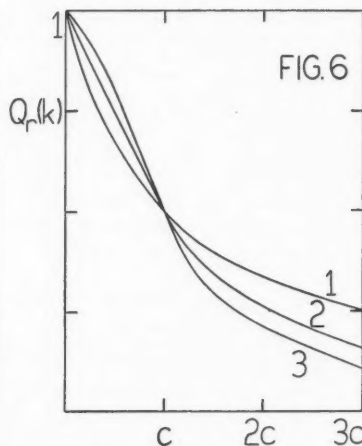
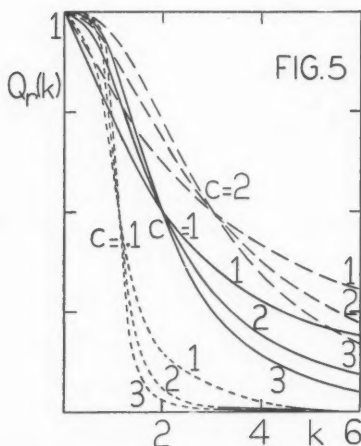


FIG. 5. Probability distribution of  $Q_r(k)$ , the probability that the signal plus noise power averaged over  $r$  samples is at least  $k$  times the averaged noise power (Equations [11], [18], [19]). The value of  $r$  is indicated on each curve.  $c$  denotes the mean signal power/mean noise power.

FIG. 6. Probability distribution of  $Q_r(k)$  when  $c \gg 1$ .

with  $u$  denoting  $k+c-1$ . The dependence of the functions  $Q_1(k) = Q(k)$ ,  $Q_2(k)$ , and  $Q_3(k)$  upon  $k$  is as shown in Figs. 5 and 6. It may be verified that for all values of  $r$  the integral [17] takes the value  $\frac{1}{2}$  when  $k = c+1$ , in agreement with the fact that the median value of  $k$  is  $(\bar{p} + \bar{p}_n)/\bar{p}$  regardless of the number of samples averaged.

The equation [8] of Section 2, which compares the signal plus noise power with the noise power at a single instant, may be used to compare the averages over a small time interval provided the noise and the signal are of the same frequency. This is approximately the case if the receiver responds to a frequency band of extremely narrow width or if the signal and the noise have a single predominant frequency. When the signal and the noise are of different frequencies let  $\omega t$  denote the increase in phase difference between the signal and noise during the time  $t$ . Suppose that  $t$  is so small that the signal and noise powers do not change significantly throughout this time. If at time  $t = 0$  the signal has power  $p$  at phase  $\theta$  in excess of the noise of power  $p_n$  then at time  $t$  the power of the signal plus noise is

$$[20] \quad q = p + p_n + 2p^{\frac{1}{2}}p_n^{\frac{1}{2}}\cos(\theta + \omega t)$$

while the average power  $Q$  over the time  $t$  is

$$[21] \quad Q = (1/t) \int_0^t q \, dt = p + p_n + 2p^{\frac{1}{2}}p_n^{\frac{1}{2}}[\sin(\theta + \omega t) - \sin \theta]/\omega t.$$

When  $\omega t = 0$  this leads to the expression [3] with  $Q$  replacing  $q$  and allows the analysis of Section 2 to proceed in terms of averages over the time  $t$ .

When  $\omega t$  is different from zero the expression corresponding to [3] is, in general, more cumbersome and the resulting analysis becomes unwieldy. However, if  $\omega t \gg 1$  the relation [21] reduces to  $Q = p + p_n$  and the probability that the average noise power lies between  $p_n$  and  $p_n + dp_n$  while the average noise plus signal power lies between  $Q$  and  $Q + dQ$  is

$$[22] \quad (1/\bar{p}\bar{p}_n)\exp[-p_n/\bar{p}_n - (Q - p_n)/\bar{p}]d p_n d Q.$$

Hence the probability that the ratio of the average signal plus noise power to the average noise power lies between  $k$  and  $k + dk$  is  $\bar{P}(k)dk$ , where

$$[23] \quad \begin{aligned} \bar{P}(k) &= (1/\bar{p}\bar{p}_n) \int_0^\infty \exp[-p_n/\bar{p}_n - (k-1)p_n/\bar{p}]p_n dp_n \\ &= c(c+k-1)^{-2}. \end{aligned}$$

$k$  is necessarily greater than unity and the probability of obtaining a ratio greater than  $k$  is

$$[24] \quad \bar{Q}(k) = c/(c+k-1).$$

If the above interval averages are again averaged over a random set of time intervals the same expressions [23] and [24] are obtained, in contrast with the case when  $\omega t = 0$ . For comparison with the results presented in Figs. 1-4 the expression [23] is plotted against  $k$  in Fig. 7 for  $c$  values of 0.1, 1, and 2, while  $\bar{Q}(k)$  is plotted in Fig. 8.

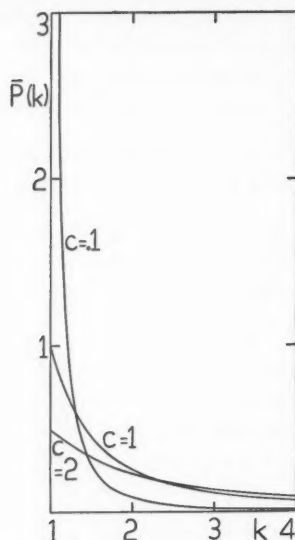


FIG. 7. Probability distribution of  $\bar{P}(k)$  (Equation [23]) applying to the case  $\omega t \gg 1$ , drawn for comparison with Figs. 1, 2, 4.

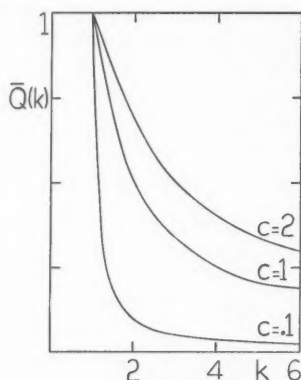


FIG. 8. Probability distribution of  $\bar{Q}(k)$  (Equation [24]) applying to the case  $\omega t \gg 1$  and drawn for comparison with Fig. 5.

#### 4. RESPONSE OF RECEIVER TO A CHANGE IN MEAN SIGNAL POWER

If, during reception of a pulse, the mean signal power changes from a value of  $\bar{p}$  at one instant to a value of  $f^2\bar{p}$  at a subsequent instant, the receiver responds to the consequent fluctuation of the power of the signal plus noise. Assuming that the phase difference between the signal and the noise does not change between the two instants, then the condition for the signal plus noise

to change from  $q$  to  $h^2q$  owing to a fluctuation of the signal power from  $p$  to  $f^2p$  is that the noise power be  $p_n$  at phase  $\theta$  in excess of the signal, where  $p_n$  and  $\theta$  are given by

$$\begin{aligned} q &= p + p_n + 2p^{\frac{1}{2}}p_n^{\frac{1}{2}}\cos\theta, \\ h^2q &= f^2p + p_n + 2fp^{\frac{1}{2}}p_n^{\frac{1}{2}}\cos\theta. \end{aligned}$$

These equations admit the solution

$$[25] \quad p_n = q(h^2 - f)/(1 - f) + pf$$

for just two values of  $\theta$  provided the condition

$$[26] \quad (1 - h)^2q \leq (1 - f)^2p \leq (1 + h)^2q$$

is fulfilled. Otherwise there is no solution for  $p_n$ .

Considering all values between  $q$  and  $q + dq$  of the noise plus signal power at the first instant and all values between  $q' = h^2q$  and  $q' + dq'$  at the second instant then, for fixed  $p$ , the end point of the vector  $p_n^{\frac{1}{2}}$  as given by equation [25] ranges over an area of

$$[27] \quad 2hqdq^{\frac{1}{2}}dq'^{\frac{1}{2}}\{4h^2q^2 - [(1 + h^2)q - (1 - f)^2p]^2\}^{-\frac{1}{2}}.$$

Thus, for fixed  $p$ , the probability that at the chosen instants the signal plus noise power lies within the above range is

$$[28] \quad (2/\pi\bar{p}_n)\exp[-(h^2 - f)q/(1 - f)\bar{p}_n - fp/\bar{p}_n]2hqdq^{\frac{1}{2}}dq'^{\frac{1}{2}}\{\dots\}^{-\frac{1}{2}}.$$

The probability of this occurring while  $p$  lies between  $p$  and  $p + dp$  is obtained after further multiplication by the factor  $(1/\bar{p})\exp(-p/\bar{p})dp$ . Putting  $dq^{\frac{1}{2}} = q^{\frac{1}{2}}dh$ , and integrating over all values of  $p$  and  $q$  within the range [26], the probability that the signal plus noise power at the second instant lies between  $h^2$  and  $(h + dh)^2$  times its value at the earlier instant is  $R(h)dh$ , where

$$[29] \quad R(h) = (2h/\pi c\bar{p}_n^2) \iint \exp[-(1 + cf)p/c\bar{p}_n - (h^2 - f)q/(1 - f)\bar{p}_n] \{\dots\}^{-\frac{1}{2}} q dq dp$$

with  $c$  denoting  $\bar{p}/\bar{p}_n$ . The integration is over all positive values of  $p$  and  $q$  for which  $(1 - h)^2q \leq (1 - f)^2p \leq (1 + h)^2q$ .

The substitution  $p = r\cos\theta$ ,  $q = r\sin\theta$ , followed by an integration with respect to  $r$  from zero to infinity, reduces equation [29] to the form

$$[30] \quad R(h) = (2hc/\pi)(1 - f)^2 \int [(1 + cf)(1 - f)\cos\theta + c(h^2 - f)\sin\theta]^{-2} \\ \times \{2(1 + h^2)(1 - f)^2\sin\theta\cos\theta - (1 - h^2)^2\sin^2\theta - (1 - f)^4\cos^2\theta\}^{-\frac{1}{2}} \sin\theta \, d\theta$$

integrated over the range  $(1 - f)^2/(1 + h)^2 \leq \tan\theta \leq (1 - f)^2/(1 - h)^2$ . Rationalization of this integral is effected by the substitution

$$[31] \quad \tan\theta = (1 - f)^2[(1 + h^2)(1 + t^2) + 4ht]/(1 - h^2)^2(1 + t^2)^{-1}$$

which reduces  $R(h)$  to the form

$$[32] \quad R(h) = 4(hc/\pi)(1 - f)^2 |1 - h^2| \int [(1 + h^2)(1 + t^2) + 4ht] \\ \times \{(1 + cf)(1 - h^2)^2(1 + t^2) + c(h^2 - f)(1 - f)[(1 + h^2)(1 + t^2) + 4ht]\}^{-2} dt$$

in which the limits of integration are  $-1$  and  $1$ . Although the integral [32] may be evaluated by elementary methods the resulting expression is, in

general, very lengthy and will not be reproduced here. For given values of  $c$ ,  $f$ , and  $h$  the integrand is a slowly varying function of  $t$  and the value of the integral may readily be estimated numerically.

The expression [32] gives the probability that a signal power fluctuation through a factor  $f^2$  will appear as a fluctuation through a factor  $h^2$  in the power of the received signal plus noise. For  $f = \frac{1}{2}$  the distributions  $R(h)$  when  $c = 0.1, 1$ , and  $2$  are as drawn in Fig. 9.

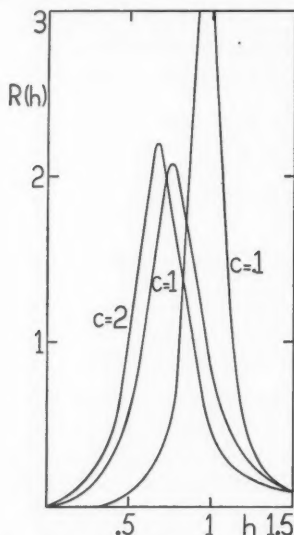


FIG. 9. Probability distribution of  $R(h)$ , the probability that a signal power fluctuation through a factor  $f^2$  will appear as a fluctuation through a factor  $h^2$  in the power of the signal plus noise. The curves are drawn for the case  $f = \frac{1}{2}$ .  $c$  denotes the mean signal power/mean noise power.

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# A MEASUREMENT AT OTTAWA OF THE CHANGE IN HEIGHT WITH LUNAR TIME OF THE *E* REGION OF THE IONOSPHERE<sup>1</sup>

By C. A. LITTLEWOOD AND J. H. CHAPMAN

## ABSTRACT

The method used by Appleton and Weekes to detect the lunar variation of height of the *E* region of the ionosphere has been used to determine the amplitude and phase of the lunar height variation at Ottawa. Observations were made from October to December, 1952. A sinusoidal variation of height of 1.5 km. amplitude and 12 hr. period was observed. The maximum height occurred about six hours after lunar transit. This result differs in phase by six hours from that observed in Cambridge in 1938.

## 1. INTRODUCTION

The variation of barometric pressure with lunar time is known at over fifty stations throughout the world. Since the form of this variation is sinusoidal and of 12 hr. period, it has been postulated that this is evidence of a tidal oscillation of the earth's atmosphere under the influence of the gravitational field of the moon. The effect is very similar to that of tides in the ocean, except that in this case the atmospheric "ocean" is not restrained by continental land masses. Theoretical investigations extending over many years have shown that the tidal pressure variation should be present in the upper atmosphere. An excellent summary of our knowledge of tidal oscillations of the atmosphere has been given by Wilkes (7).

The energy in the tidal oscillation at the ground is very small, the oscillation having an amplitude of the order of 0.01 mm. In the upper atmosphere the relative pressure change is much greater, so that the effect is easier to observe. Pressure oscillations in the upper atmosphere are indicated by variations in the ionized layers, including variation of heights of the layers.

The lunar variation in height of the *E* region was demonstrated by Appleton and Weekes in 1938 (1). They showed that a sinusoidal change in height of 12 hr. period was present in the *E* layer of the form

$$0.93 \sin(2t + 112^\circ) \text{ km.,}$$

as measured at Cambridge, England (where  $t$  is the local lunar hour angle).

Using the now common radio pulse methods, they measured the height of the *E* layer to the nearest half kilometer, at a frequency of 1.8 Mc./s., for 11 periods each of 12 to 14 days. The average daily variation for each period due to solar ionization was removed, and the residual data were examined for the lunar effect. Various statistical tests applied to the results showed beyond any doubt that the sinusoidal variation, of 12 hr. period according to lunar time, was real, and that variations of other periodicities were unimportant.

Following this work, Martyn observed a lunar variation in *E* region heights

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of the same form at Canberra and Brisbane (6). His data were obtained from the examination of records covering a period of four years from regular ionosphere sounding stations. At these stations the amplitude of the variation was smaller, and the phase was in opposition to, that is, different by six hours from, the Cambridge results. Martyn suggested that the cause of this difference was due to motion of the ionosphere across the earth's magnetic field. A consequence of this suggestion is that the phase of the lunar variation of height of the *E* layer should change by six hours at about  $35^\circ$  latitude.

This paper describes a measurement of the lunar variation of the *E* region height at Ottawa, Canada ( $45.3^\circ$  N.,  $76^\circ$  W.) using the method of Appleton and Weekes.

## 2. EXPERIMENTAL METHOD

The equipment used in the experiment was conventional. A pulse transmitter operated on a frequency of 1.8 Mc./s., with a peak power output of 15 kw. and a pulse width of 70  $\mu$ sec. Non-resonant delta antennas which radiate in a predominantly vertical direction were used for both transmitting and receiving. The receiver was a commercial type modified for pulse reception.

A modified Loran cathode ray tube display was used for accurate determination of the delay time of the echo from the *E* region of the ionosphere. The positions of the transmitted pulse and the received signal were measured at the peak of the pulse in both cases. In the case of the transmitter pulse, this involved reducing the receiver gain until the transmitted pulse was about the same height as the received pulse on the cathode ray tube. Readings were made manually every 15 min. through the day.

The accuracy of the measured heights was checked periodically by comparing the delay times of the once-reflected and twice-reflected echoes. The measurements were self-consistent to within one kilometer of equivalent height.

## 3. REDUCTION OF THE DATA

Observations were made every 15 min. from 08:00 to 16:00 hr. local standard time, or over as long a period as ionospheric conditions permitted. The critical frequency of the *E* region was measured at the same time as each height determination made at 1.8 Mc. Observations were made during the following periods:

Oct. 15-29, 1952,  
Nov. 13-26, 1952,  
Nov. 27-Dec. 24, 1952.

Following the method of Appleton and Weekes, all height observations which were made when the critical frequency was less than 2.4 Mc. were rejected. This procedure ensures negligible retardation of the pulse in the ionospheric region and an equivalent or observed height which is nearly the true height. Anomalous heights were sometimes observed which appeared to indicate reflections from a layer below the main layer. These readings, too, were rejected.

For each of the three periods the average height for each quarter hour was plotted and a smooth curve was drawn through the points. The curve for the first period is shown in Fig. 1. This is the solar control or the diurnal variation

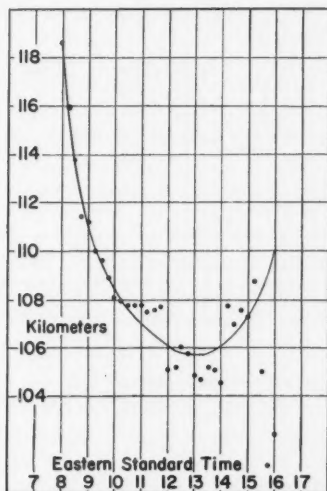


FIG. 1. Average quarter-hourly heights of the *E* region measured during period of Oct. 15-29, 1952.

in height of the layer caused by the changing zenith angle of the sun. In order to isolate the small lunar effect it was desirable to first remove the large solar effect, and therefore this solar curve was subtracted from the heights measured in the first period. The residuals were grouped according to the hour angle of the moon, e.g., all residuals corresponding to lunar hour angles 23:30 to 00:30 and 11:30 to 12:30 hr. were grouped as belonging to lunar hour angle 0. The average was determined for each lunar hour and the semidiurnal component was found by harmonic analysis. This was done for each of the three periods and the results are displayed on a harmonic dial in Fig. 2. The center of gravity of these points corresponds to a sine curve represented by

$$h = 1.46 \sin(2t - 81^\circ),$$

where  $h$  is in kilometers and  $t$  is the lunar hour angle. An allowance of 1% for the grouping of the data (3) has been made.

We may take this as the best value of the lunar semidiurnal component from the data of this experiment.

#### 4. DISCUSSION

The scatter of points on the harmonic dial of Fig. 2 is considerably greater than on the corresponding diagram given by Appleton and Weekes. The cause is probably to be found in the amount of selection of the data made before

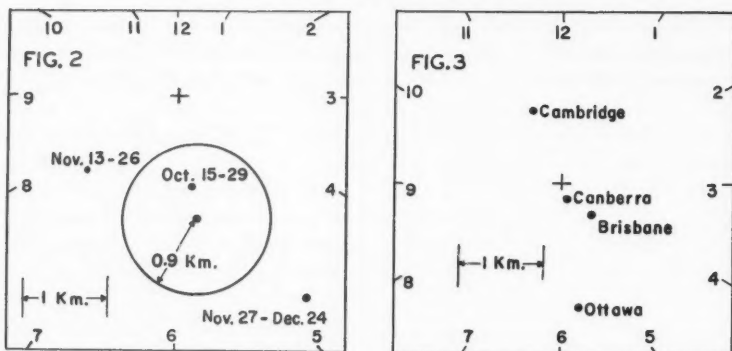


FIG. 2. The lunar semidiurnal component in the height of the *E* region at Ottawa, 1952. The time of maximum amplitude is given by the angular position of each point. The magnitude is shown by the distance from the dial center.

FIG. 3. The lunar semidiurnal components in the height of the *E* region in England, Canada, and Australia.

analysis. Since in our case the measurements were not made from a continuous record, but, rather, were the instantaneous measurements of height made at fixed times, it was difficult for us to identify those sudden changes in height which are probably due to transient subsidiary ionization maxima in the layer. In the light of these remarks, the results shown in this diagram appear to give a significant measure of the lunar variation of height.

For comparison, the measurements reported by Appleton and Weekes for Cambridge, and by Martyn for Canberra and Brisbane, are given on Fig. 3, along with the result reported in this paper. It can be seen that the amplitudes at Cambridge and Ottawa are comparable, as one might expect from their relative proximity in geographic latitude. The phase of the Ottawa determination however is nearly the same as that for Canberra and Brisbane, and in opposition to that for Cambridge.

This difference in phase at Ottawa is somewhat surprising, since Martyn's theory based on the motion of ionized gas across the earth's magnetic field predicts that the change of phase should occur at  $35^\circ$  latitude (5). This apparent anomaly warrants further investigation, possibly through a series of measurements along a chain of stations running from north to south through the middle and upper latitudes. It is also evident that the determination of the lunar tidal variation at Ottawa should be repeated at other times of the year.

##### 5. APPENDIX, TESTS OF SIGNIFICANCE

The lunar variations in height which were sought in this experiment were small compared with variations due to other causes. Hence it is necessary to show that the result is statistically significant. Two tests were used, namely Fisher's analysis of variance (4) and Bartels' harmonic dial test (2).

###### 1. Analysis of Variance

The following symbols will be used:

$y$  = any observed height less the solar curve,

$y_p$  = any observed height at lunar hour  $p$ , less the solar curve,

$n_p$  = the number of observations at lunar hour  $p$ .

This test was applied to all the data. The variance of the  $y_p$ 's about the hourly means,  $\bar{y}_p$ , is

$$\frac{\sum \sum (y_p - \bar{y}_p)^2}{\sum n_p - 12} = \frac{21921.71}{(821 - 12)} = 27.097.$$

This is a measure of the scatter in the readings due to all causes other than lunar variation.

The variance of the hourly means,  $\bar{y}_p$ , about the over-all mean,  $\bar{y}$ , is

$$\frac{1}{11} \sum n_p (\bar{y}_p - \bar{y})^2 = \frac{1657.87}{11} = 150.72.$$

If there were no effect depending on lunar hour the two variances would be the same. As the second variance is much larger a real lunar effect is suggested. We now determine the probability that the second variance could be this large by chance.

Following Fisher, we define  $Z$  as half the difference of the natural logarithms of the variances,

$$\begin{aligned} Z &= \frac{1}{2}(\log_e 150.72 - \log_e 27.097) \\ &= 0.86. \end{aligned}$$

The probability of getting this large a  $Z$  by chance is less than 0.1%. Hence there appears to be a significant effect dependent upon lunar hour.

Now we determine the probability that the points could lie as closely as they do to the harmonic curve by chance. First we find the correlation coefficient (3),  $r$ , of all the points with the harmonic curve:

$$r = R/\sqrt{2} \delta,$$

where  $R$  = amplitude of harmonic curve = 1.46,

$\delta$  = standard deviation of all the observations = 5.36.

Hence  $r = 0.19$ .

Following Fisher, we write

$$\begin{aligned} Z &= \frac{1}{2} \log_e [(1+r)/(1-r)] \\ &= 0.195. \end{aligned}$$

The standard deviation of  $Z$  is given by

$$\sigma_Z = 1/(n_p - 14)^{\frac{1}{2}} = 1/28.4.$$

Fisher shows that  $Z$  is nearly normally distributed. The probability of obtaining by chance a value of  $Z/\sigma_Z = 5.54$ , by reference to a table of the probability integral, is seen to be less than  $10^{-6}$ . From this test we conclude that a significant harmonic of 12 hr. period according to lunar time is present in the data.

## 2. Harmonic Dial Test

From the data exhibited in Fig. 2 we can test whether or not the tidal variation persists over the period of observation. The co-ordinates of the center of gravity of the points corresponding to the three periods are:

$$x_0 = \frac{1}{3}\sum x_v \text{ and } y_0 = \frac{1}{3}\sum y_v.$$

The probable error circle, which has been drawn in Fig. 2, is centered at  $x_0, y_0$  and has a radius

$$P = 0.833 \left\{ \frac{1}{3} [\sum (x_v - x_0)^2 + \sum (y_v - y_0)^2] \right\}^{\frac{1}{2}} \\ = 0.90.$$

The mean amplitude is  $(x_0^2 + y_0^2)^{\frac{1}{2}} = 1.45$ . Let the ratio of the mean amplitude to the probable error radius be  $k = 1.45/0.90$ . Then  $(\frac{1}{2})^{k^2} = 0.166$  is the probability that a point will fall outside a circle centered at  $x_0, y_0$  and passing through the origin. The harmonic dial test shows that an apparent lunar semidiurnal variation persists over the period covered by the data.

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## THE RUBIDIUM TRANSITION AT $\sim 180^\circ\text{K}$ .<sup>1</sup>

By F. M. KELLY<sup>2</sup> AND W. B. PEARSON

### ABSTRACT

Resistivity measurements have shown the presence of a "transition" in rubidium at  $\sim 180^\circ\text{K}$ . The nature of this transition has been further investigated by dilatometric and X-ray studies of thermal expansion, thermoelectric force measurements, and X-ray examination for the possibility of a change of crystal structure. Evidence for the transition in properties other than resistance measurements is very variable and, although the transition does not appear to involve a change of crystal structure, its characterization as an electronic change remains obscure. Analysis of resistance measurements shows that the resistive behavior of rubidium above  $180^\circ\text{K}$ . must be regarded as anomalous.

### INTRODUCTION

Electrical resistance measurements of rubidium made continuously from  $\sim 300^\circ$  to  $4^\circ\text{K}$ . (10) during a systematic investigation of the electrical properties of the alkali metals revealed the presence of a relatively sharp "transition" in the resistance variation at  $\sim 180^\circ\text{K}$ ., Fig. 1. The characteristic temperatures,  $\theta$ , determined from the resistance measurements (cf. (11)) suggest that the entire resistance variation above the transition point must be regarded as abnormal. Below the "transition" the computed value of  $\theta$  agrees rather well with that to be expected from other properties; for example the estimate from the Lindemann melting point formula is  $\sim 57^\circ\text{K}$ . Above the transition however a quite unreasonable  $\theta$  ( $\sim 350^\circ\text{K}$ .) emerges and we appear forced to assume the intrusion of some additional mechanism of electron scattering which is not envisaged in the theoretical model (due to Bloch, Fröhlich, Wilson, Mott, and others) of scattering by thermal lattice vibrations characterized by  $\theta$ .

We have consequently been led to study other physical properties in the hope of determining the nature of this anomaly, which may arise either from a change of lattice structure or from some specific change in the properties of the electrons responsible for conduction, although, of course, electrons and lattice are never free from mutual interaction, despite the convenient ideal of "free" electrons. Dilatometric, X-ray, and microscopic investigations have been made to look for any change of lattice properties, and thermoelectric measurements have been made to supplement the resistance measurements, although thermal conductivity and magnetic susceptibility measurements would also be of interest. The high reactivity of rubidium with oxygen made it mandatory to prepare all specimens by running the molten metal under vacuum into glass tubes. As the rubidium wets and adheres to the walls of these tubes the question arises whether the large contraction on cooling may either inhibit, or lead to, a crystal structure change or perhaps introduce stacking-faults with consequent change of physical properties. The rather variable dilatometric and X-ray expansion coefficients (and also specific heat,

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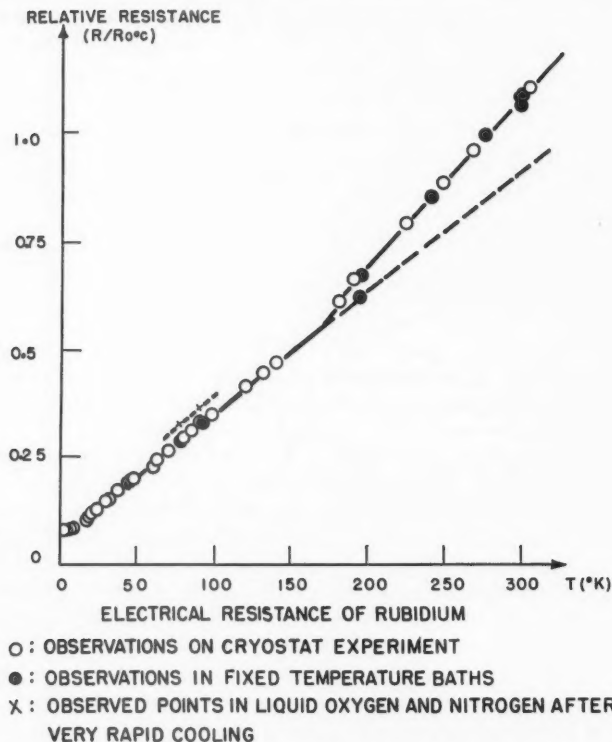


FIG. 1. Taken from *Experiments on the Electrical Resistivity of Metals* by D. K. C. MacDonald, Phil. Mag., Ser. 7, 43: 479. April 1952.

cf. (4), measurements) may possibly provide evidence for this, although it is in any case inherently difficult to make very precise measurements of the expansion of rubidium dilatometrically or by X-rays.

The rubidium used throughout these investigations was supplied by A. D. MacKay, New York, and had a residual resistance ratio,  $R_{4.2^\circ}/(R_{273.2^\circ} - R_{4.2^\circ}) \sim 23 \times 10^{-3}$ . Spectroscopic analysis carried out in the laboratory showed the presence of some sodium and traces of cesium and calcium.

#### DILATOMETRIC MEASUREMENTS

The thermal expansion of rubidium was investigated over the complete range from 120° to 295°K. in a glass dilatometer placed in a stirred isopentane bath, which was cooled progressively by addition of solid carbon dioxide or liquid nitrogen. The dilatometer, whose walls had been coated with liquid paraffin to prevent the rubidium adhering, was filled with molten metal under vacuum and then sealed off. About one gram of metal was used which nearly filled the dilatometer bulb. The remaining space was then filled with isopentane



by removing the tip of the capillary tube in a bath of isopentane, and the level in the capillary adjusted for convenient observation with a cathetometer. The height of the isopentane column was then measured directly as a function of temperature. Temperatures were measured with a calibrated copper-constantan thermocouple placed close to the dilatometer bulb.

The behavior of the rubidium was found to be very variable. On the first cooling run the variation with temperature of the column-height, which was found to be linear, showed a change of slope at 188°K. When the run was repeated the change of slope took place at 210°K., and on a third cooling run no anomaly was observed down to 134°K. The dilatometer was then further cooled so that the isopentane froze in the capillary and on subsequent warming up a change of slope occurred at 148°K., but no anomalies whatever were observed in two final runs when the specimen was cooled to ~130°K.

A second dilatometer was made and filled, but on two coolings to ~130°K. no transition was found. The rubidium metal was then transferred to a suitable mold for measurement of resistance and transitions similar to those observed by MacDonald at ~180°K. were again observed.

Two "blank" runs made with the dilatometer containing isopentane only revealed no anomalous behavior.

The ratio of expansion coefficients above and below the change of slope (when observed) has been reliably determined as 4.7, although uncertainties of some 25% in the estimated volume of rubidium and the presence of a small volume of isopentane in the bulb make estimates of the absolute coefficient of volume expansion rather uncertain. Above the transition it is estimated as  $3 \times 10^{-4}$  and below as  $6 \times 10^{-5}$ , the uncertainty in each case being some 40%. The expansion coefficients appear to be very closely independent of temperature in the range studied.

#### X-RAY INVESTIGATION

No definite evidence of a change of crystal structure has been found. However, the investigation suffers inherently from the difficulty already discussed due to the rubidium wetting and adhering to the walls of the glass capillaries.

Specimens for X-ray examination were prepared by vacuum melting the metal into a fine, thin-walled glass capillary which formed part of the glass melting tube. The metal was then slowly cooled or quenched in liquid nitrogen according to the type of crystals desired. Some single crystal photographs were taken in the Unicam S-25 goniometer while Debye-Scherrer photographs were taken in a camera built to use the Unicam 19 cm. film cassette. Details of the camera and its operation have already been described (14).

It is difficult to obtain satisfactory photographs of rubidium because of strong absorption and a large temperature-factor in the room-temperature region. Development of preferred orientation and fragmentation of the crystals on cooling to low temperatures also cause trouble. Using copper radiation the highest reflection obtained at room temperature is (321), (Bragg angle,  $\theta \sim 30^\circ$ ), but at 77°K. we have obtained resolved doublets extending to (550, 710), ( $\theta \sim 76^\circ$ ). Lattice parameters have been calculated in true Ångströms using  $\lambda_{\text{CuK}\alpha_1} = 1.54050 \text{ Å}$ .

## LATTICE PARAMETERS OF RUBIDIUM

Table I shows the lattice parameters obtained for rubidium during the present work by extrapolation of resolved doublets against the function  $\frac{1}{2}(\cos^2 \theta / \sin \theta + \cos^2 \theta / \theta)$  proposed by Nelson and Riley (13), and also the

TABLE I  
RUBIDIUM LATTICE PARAMETERS

Rubidium specimen	77°K.	87°K.	Temperatures		291°K.	297°K.
			90°K.	263°K.		
Rubidium III	5.612 Å		5.619 Å			5.700 Å*
Rubidium IV	5.609 Å		5.616 Å			5.697 Å*
Rubidium VI	5.607 Å					
Simon and Vohsen (16)		{ 5.62 kX 5.63 Å				
Böhm and Klemm (1)				{ 5.66 kX 5.67 Å		
Hume-Rothery and Lonsdale (8)			{ 5.624 kX 5.635 Å		{ 5.699 kX 5.710 Å	

\*Estimated from room temperature photographs using slope of 'a' against  $\frac{1}{2}(\cos^2 \theta / \sin \theta + \cos^2 \theta / \theta)$  obtained from photographs at 77° and 90°K., all three photographs being taken with the specimen undisturbed so that the eccentricity error is the same for each.

lattice parameters determined by previous workers. In all earlier work lattice parameters have been obtained by averaging or by extrapolation of reflections extending only to a Bragg angle of  $\sim 30^\circ$ . The lattice parameters at 77°K., obtained by extrapolation of resolved doublets extending to a Bragg angle of  $\sim 76^\circ$ , have an accuracy approaching 1 in 5000 although a variation of some 0.1% is found between specimens. This variation between specimens is much less than the difference between our value at 90°K. and that found by Hume-Rothery and Lonsdale (8). However, Hume-Rothery and Lonsdale's value at 90°K. was obtained by calculation using their expansion coefficient precisely determined from photographs at 90° and 291°K. and a lattice parameter of 5.699 kX at 291°K. obtained by extrapolation of reflections extending only to a Bragg angle of  $\sim 30^\circ$ .

## THERMAL EXPANSION COEFFICIENT

The characteristic temperature of rubidium is  $\sim 60^\circ\text{K.}$  at low temperatures so that the expansion coefficient is expected (and observed) to show approximately linear variation with temperature in the region 120–300°K., while there may be an appreciable increase just below the melting point due to development of vacancies in the lattice. Five series of photographs were taken between room temperature and 120°K. either at progressively lower temperatures or at random. These lead to the average expansion coefficients shown in Table II. Because only low angle reflections are obtained at high temperatures we have calculated expansion coefficients from 120° to 300°K. from the change of specific interplanar spacings with temperature on photographs usually made at  $\sim 30^\circ$  intervals. This treatment is valid provided that the specimen is not disturbed throughout the series of photographs so that centering and absorp-

TABLE II  
 AVERAGE LINEAR EXPANSION COEFFICIENTS OF RUBIDIUM

Rubidium specimen	Temperature range	(110)	(200)	Reflections (211)	(220)	(310)	(321)
Rubidium III	120-240			58			54
Rubidium III (later)	160-300			51			54
Rubidium V	170-300		75	77			77
Rubidium VI	130-300	76	75		75	75	
Rubidium VI (later)	120-200				61	61	
Rubidium III	77-300	70	} Estimated from photographs at 77°K., 90°K., and room temperature by use of (211) and (321) reflections and also from extrapolated parameters				
Rubidium IV	77-300	67					
Hume-Rothery and Lonsdale (8)	176-291	66	} Macroscopic measurements				
Hackspill (7)	273-310	90					
Deuss (5)	275-291	~90					

tion errors affecting the reflections from a particular plane remain the same. No definite evidence of an inflection was observed in the neighborhood of the resistivity transition temperature of 180°K. or at a lower temperature.

Table II shows that an acceptable value of  $\alpha$  is probably  $66 \times 10^{-6}$  between 77° and 300°K. in agreement with Hume-Rothery and Lonsdale (8). The variation found between different specimens, or between different runs in the same specimen, *greatly* exceeds the relative self-consistency of the results for any particular run. These variations are reminiscent of the random behavior found in the dilatometric experiments but they may result from deformation on cooling the metal which adheres to the glass capillary wall.

#### MICROSCOPICAL INVESTIGATION

The method of Rosenhain and Murphy (15) was used to examine the *surface* of rubidium above and below the transition temperature. Because of the extremely reactive nature of the metal we had to retain the thin glass covering the surface during observations. On the first cooling, some of the rubidium withdrew from the glass surface and it was possible then to observe both the free and attached rubidium surfaces at temperatures above and below the expected transition temperature by cooling the specimens with liquid nitrogen. No suggestion of a structural change was found on cooling down to 77°K.

#### THERMOELECTRIC MEASUREMENTS

Thermoelectric measurements were also undertaken since any major change in the degree of freedom of the conduction electrons produced by the transition would be expected to make itself evident in the thermoelectric power. However, only gradual variation of thermoelectric power was found throughout the whole range investigated and no anomalies were apparent such as might be associated with the transition.

Measurements of the thermoelectric force of a specimen some 6 cm. long were made when a temperature difference of 3° to 4° C. was maintained between the ends. The temperature gradient was obtained by packing the

specimen mold in glass wool in a suitable test tube placed in a beaker of isopentane set in a large Dewar flask. Cooling the bottom of the isopentane bath with liquid nitrogen set up a temperature gradient in the sample, while constantan heaters wound around the ends of the sample aided in adjusting the temperatures.

Specimen molds were made of a 1.5 mm. diameter capillary tubing with short lengths of thin-walled platinum tubing sealed in the ends. The rubidium was melted under vacuum and run into the capillary tube which was then sealed off. A platinum and a constantan wire were then soldered (as a thermojunction) to the platinum tubes at each end of the specimen, thus ensuring very close thermal contact with the rubidium. All potentials were continuously recorded on a recording potentiometer after amplification with a d-c. microvolt amplifier. Measurements were then repeated with the temperature gradient in the specimen reversed, so that thermoelectric errors could be eliminated, and in particular the measured temperature interval did not depend on the assumption that the thermocouples were identical. The thermoelectric power of rubidium measured against platinum showed no discontinuity in the region of 180°K. although there was a gradual change in slope (and hence in the Thomson coefficient).

Thermoelectric force measurements were also made in the Collins helium cryostat using a 5 ft. 6 in. long capillary column of rubidium against lead as a reference metal. One junction was maintained in melting ice outside the cryostat and the other was bonded to the bulb of a gas thermometer in the main well of the liquefier (for further description of the method see MacDonald and Pearson (12)). The thermoelectric force was measured over the range from  $\sim 4.2^\circ\text{K.}$  to  $\sim 300^\circ\text{K.}$  and no anomaly was observed in the region of the resistive "transition".

#### DISCUSSION

The rubidium transition may be compared with that found in chromium at  $\sim 37^\circ\text{C.}$  as in neither case does a crystal structure change appear to be involved. The chromium transition, which has recently been thoroughly investigated and reviewed by Fine, Greiner, and Ellis (6), takes place over a range of temperature without hysteresis. Electrical resistivity measurements have been interpreted as an electronic transition with increase in the ratio of  $4s$  to  $3d$  electrons on proceeding to higher temperature (Söchtig (17)). The almost constant paramagnetism throughout the range suggests that the transition is not due to loss of antiferromagnetism (6). The transition does not influence the heat capacity of chromium.

The rubidium transition by contrast appears to be very variable depending in most cases on specimen history. The only property in which it has been consistently found at the same temperature is in measurement of electrical resistivity. The transition appears to be without influence on the thermoelectric power, but appears quite significantly in heat capacity measurements (Dauphinee and Preston-Thomas (4)).

Consideration of the resistivity change in rubidium shows that the behavior

must be regarded as normal *below* the transition temperature. An electronic transition of  $5s$  to  $4d$  on proceeding to higher temperatures (although giving increased resistivity) seems generally unlikely, while the change in the characteristic temperature  $\theta$  is inexplicable.

No definite evidence has been found to link the transition with the presence of impurities and this point has been carefully examined as regards gaseous impurities (with the exception of oxygen) in the heat capacity measurements of Dauphinee and Preston-Thomas. The presence of very little oxygen in cesium leads to a eutectic reaction at  $271^\circ\text{K}$ . (3) but it seems very unlikely that a similar contamination in rubidium could lead to a eutectic reaction at a temperature as low as  $180^\circ\text{K}$ .

The nature of the transition in rubidium therefore remains unsolved, largely owing to the great reactivity of the metal with oxygen and water, which makes it difficult to conduct precise measurements except when the metal is sealed in a glass tube. In crystal structure investigations this is particularly so, and although no evidence of structural change has been found at present, the possibility cannot be entirely ruled out until observations have been made on rubidium which is not physically enclosed in a glass capillary, for it is known that this physical enclosure of a specimen may inhibit or greatly reduce the extent of a diffusionless type of transition (Bowles (2)). On the other hand evidence from the Lindemann melting point formula strengthens the belief that no lattice change takes place. The value of the characteristic temperature,  $\theta$ , calculated from the melting point,  $\sim 57^\circ\text{K}$ ., is in good agreement with the nominal value of  $\theta$  obtained at low temperatures from resistance measurements, indicating that the lattice has remained unchanged during the "transition" and up to the melting point. By contrast the value of  $\theta$  ( $\sim 350^\circ\text{K}$ .) (cf. Kelly and MacDonald, Method 3 (9)) obtained from resistance measurements above the transition is clearly anomalous and shows that the change is accompanied by a marked progressive increase of scattering or corresponding decrease in the number of conduction electrons such as might occur in the course of an electronic transition.

*Note added in proof:* More recent investigations presently in progress on the electrical resistance of rubidium, which had been pumped continuously in the molten state for about two weeks during the specific heat investigation (Dauphinee and Preston-Thomas (4)), lend some further weight to the suggestion that the "transition" may be due to the presence of a chemical impurity such as, for instance, oxygen.

#### ACKNOWLEDGMENT

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# THE HYPERFINE STRUCTURE OF MERCURY EXTRACTED FROM NEUTRON-IRRADIATED GOLD<sup>1</sup>

BY R. E. BEDFORD<sup>2</sup> AND A. M. CROOKER

## ABSTRACT

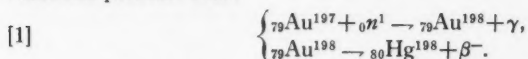
The h.f.s. of suitable lines excited in mercury extracted from several gold samples in different neutron fluxes has been photographed with a Fabry-Perot interferometer. As well as the strong lines due to Hg<sup>198</sup>, weak lines are also observed for Hg<sup>199</sup> owing to the Au<sup>198</sup> capturing a neutron before the  $\beta$ -decay. The branching ratio is

$$\frac{\text{Au}^{198}(\text{capture})}{\text{Au}^{198}(\text{decay})} = \frac{\text{Hg}^{199}}{\text{Hg}^{198}} = \frac{\sigma F}{\lambda}.$$

Using the measured ratio of Hg<sup>199</sup> to Hg<sup>198</sup>,  $\tau = 2.69$  days, and the neutron fluxes we have determined for the Au<sup>198</sup> capture cross section,  $\sigma = (1.78 \pm 0.10) \times 10^4$  barns.

## INTRODUCTION

During the past century many attempts have been made to find a suitable standard of length, and for years the wavelength of the cadmium red line has been used for that purpose. The green radiation of mercury was originally discarded because the line exhibited too complex a structure due to the presence of seven stable isotopes. However, the extraction of mercury from neutron-irradiated gold, announced first by Wiens (16), has overcome this difficulty, as emphasized and exploited by Meggers (10, 11, 12). When the single natural gold isotope, Au<sup>197</sup>, undergoes neutron bombardment the reactions proceed thus:



The half-life of the associated  $\beta$ -decay is 2.69 days. The mercury may then be removed from the gold by heating to 450° and condensing the evaporated metal. Initially the yield of mercury using this process was insufficient for the manufacture of lamps, but the higher neutron fluxes available following World War II eliminated the objection.

Hg<sup>198</sup>, being an even-even nucleus, has no spin or magnetic moment and so shows single, sharp spectral lines. The early workers (1, 9) found that the mercury prepared as above did not however exhibit single lines—that in fact variable amounts of <sup>80</sup>Hg<sup>199</sup> were also present. The latter confuses the line pattern because of its hyperfine structure ( $I = \frac{1}{2}$ ;  $\mu = +0.4993$  n.m.). The origin of the <sup>80</sup>Hg<sup>199</sup> was first attributed to neutron capture by traces of <sup>78</sup>Pt<sup>198</sup> as an impurity in the gold. Hill and Mihelich (7) suggested that the Hg<sup>199</sup> arose from neutron capture by the Au<sup>198</sup>, thus:

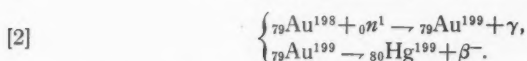
<sup>1</sup>Manuscript received October 8, 1954.

Contribution from the Department of Physics, University of British Columbia, Vancouver, B.C.

This paper was presented by Dr. G. M. Shrum to the Annual Meeting of the Royal Society of Canada at Winnipeg as a contributed paper, #90, Section III. It is based on an unpublished thesis presented by one of us (R. E. B.) to the Department of Physics, University of British Columbia, in partial fulfillment of the requirements for the degree of Master of Arts.

<sup>2</sup>Holder of a bursary under the National Research Council of Canada, 1952-53.





The  $\beta$ -decay half-life is 3.3 days. For equations [2] to have a measurable influence the neutron capture cross section of  ${}^{79}\text{Au}^{198}$  must be extremely large. Hill and Mihelich estimated the cross section to be  $3.5 \times 10^4$  barns from a comparison of the intensities of the 0.159 Mev.  $\gamma$ -ray of  $\text{Au}^{199}$  and the 0.411 Mev.  $\gamma$ -ray of  $\text{Au}^{198}$ . In a later paper (6) Hill suggests that the value  $1.6 \times 10^4$  barns is more appropriate.

The cross section may also be obtained from a determination of the ratio in which  $\text{Hg}^{199}$  and  $\text{Hg}^{198}$  are produced. During neutron irradiation a  $\text{Au}^{198}$  nucleus may either (1) undergo  $\beta$ -decay or (2) capture a neutron. The respective probabilities of these events occurring per second are for process (1)  $\lambda$ , where  $\lambda$  is the disintegration constant, and for process (2)  $\sigma F$ , where  $\sigma$  is the neutron capture cross section and  $F$  is the neutron flux in neutrons per  $\text{cm}^2$  sec. We should therefore expect to obtain a sample of mercury containing the isotopes in the ratio

$$[3a] \quad \text{Hg}^{199}/\text{Hg}^{198} = \sigma F/\lambda,$$

providing there is no appreciable separation of the isotopes in the distillation process. The ratio  $\text{Hg}^{199}/\text{Hg}^{198}$  may be measured from an interferometric study of the h.f.s. of the mercury spectral lines. Since  $\lambda$  and  $F$  are known, we have

$$[3b] \quad \sigma = (\lambda/F) \text{Hg}^{199}/\text{Hg}^{198}.$$

An accurate knowledge of  $\sigma$  will allow one to state the maximum allowable neutron flux density to be used in the production of mercury for ultimate wavelength standards.

#### EXPERIMENTAL PROCEDURE

##### (a) *Extraction of the Mercury*

The gold, which was in the form of ribbon 0.012 in. thick wound on thin aluminum wire, was still highly radioactive owing to a slight trace of silver ( $\text{Ag}^{110}$  has a 270-day half-life and decays by  $\beta$ -emission to  $\text{Cd}^{110}$ ).<sup>\*</sup> The mercury was present as a volume distribution throughout the gold and was extracted using the apparatus shown in Fig. 1. the furnace tube  $AB$  of vycor and source tube  $C$  of transparent silica were joined to the trap and pumping line by a graded seal. The system was thoroughly cleaned and the furnace tube inserted into an electrically heated furnace. A temperature of  $500^\circ\text{C}$ . was maintained for a period of eight hours (under evacuation), the source tube being torched at intervals to drive off absorbed water vapor. Helium was admitted to the system and pumped off—this flushing procedure was repeated several times to carry off any trapped gases. With the helium pressure raised slightly above atmospheric the tubulation  $B$  was broken off, the irradiated gold sample introduced into the furnace tube, and the opening  $B$  resealed.

<sup>\*</sup>We are indebted to Dr. D. B. James and Mr. K. A. Laurie of the nuclear physics group for the measurement of the  $\gamma$ -rays at 648,900 and 1440 kev., which led Dr. Reed Johnston of Suffield to identify the radioactive contaminant as 270-day  $\text{Ag}^{110}$ .



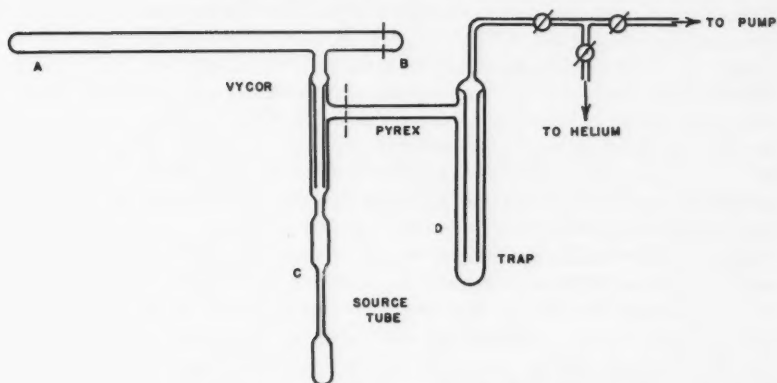


FIG. 1. Apparatus for extraction of mercury.

The distillation of mercury was begun in a helium pressure of about 1 mm. with the trap and source tube immersed in liquid air. The gold was kept at a temperature of  $450^{\circ}\text{C}$ . for five hours while the escaping mercury vapor condensed on the cold walls of the source. After the evaporation the outer parts of the system were again flamed to drive all condensed mercury into the source tube. Water vapor, introduced with the gold sample and condensed in the source tube, was evaporated by allowing the system to stand at room temperature for several minutes. A final flushing with helium was performed, the pressure pumped down to about 10 microns, and the source tube sealed off.

This tube was then set up as an electrodeless discharge source. The excitation was accomplished using a radiofrequency oscillator similar to that described by Harnwell (5). When water cooled, the source is admirably adapted for high resolution spectroscopy. Very little trouble was experienced in producing a stable discharge of high intensity. Extensive investigations by Meggers and Westfall (12) and Jacobsen and Harrison (8) have shown that the life of such tubes is dependent on the frequency of the exciting oscillation, the lifetime increasing with increasing frequency. After a period varying from 30 to 300 hr. the mercury is apparently driven into the walls of the tube. Providing no absorbed gases are present the source may be rejuvenated by heating with a torch.

#### (b) Spectroscopic Procedure

Sixteen Hg I lines were photographed by projecting Fabry-Perot interference fringes on the slit of a Hilger E-1 quartz spectrograph using a quartz fluorite achromat of 50 cm. focal length. The silica plates of the etalon had aluminum reflecting surfaces with individual reflecting coefficients of 0.805. The etalon spacer was 0.9772 cm. in length.\* Of the sixteen lines photographed, four were

\*The Hg I wavelengths reported in Table II required a spacer length of 0.9772015 cm. to yield consistent values for the observed fractional parts.

chosen as having the most suitable h.f.s. patterns for intensity measurements—namely those at  $\lambda\lambda$  5461, 4077, 3341, and 3125 Å. By “a suitable pattern” is meant one in which the strongest component of the  $\text{Hg}^{199}$  h.f.s. (i) may be identified unambiguously without neighboring satellites or overlapping orders, and (ii) lies midway between the more intense  $\text{Hg}^{198}$  lines. Density marks were also photographed on each plate using a rotating step sector giving relative intensities of 1, 2.08, 7.50, 21.56, 83.86, 201.68. Exposure times for the h.f.s. patterns varied from 2 to 10 min. Eastman II-F(3) plates were used, and were developed for six minutes in D-19 and fixed in F-5.

The intensity measurements of the h.f.s. were carried out using a Moll microphotometer in conjunction with an automatic recording Brown potentiometer. Two types of radiation detectors were used—a thermocouple with d-c. recording and a lead sulphide cell with chopped beam and a-c. recording. The former was subject to considerable drift (found to be quite linear) whereas the latter, although more sensitive, was found to introduce more noise. The reproducibility by both detectors was excellent. The heights of the maxima on the photometric trace were measured to within 1/60 in.

#### RESULTS

Twenty-one independent determinations of the ratio  $\text{Hg}^{199}/\text{Hg}^{198}$  were made from the four lines on six different plates. The results are summarized in Table I. We get as a mean value

$$\text{Hg}^{199}/\text{Hg}^{198} = 0.274 \pm 0.015.$$

The probable error quoted includes only the statistical fluctuations.

TABLE I  
DETERMINATION OF RATIO  $\text{Hg}^{199}/\text{Hg}^{198}$

$\lambda(\text{\AA})$	$k^*$	No. of plates	Ratio $\frac{\text{Hg}^{199}}{\text{Hg}^{198}} \uparrow$		Mean ratio per line
5461	5/2	5	0.238 0.273 0.270	0.287 0.251	0.258
4077	3/2	5	0.242 0.235 0.257	0.246 0.310	0.258
3341	5/2	5	0.310 0.331 0.322	0.295 0.286	0.309
3125	3	6	0.257 0.267 0.252	0.340 0.241 0.271	0.271
Mean ratio			0.274		0.274

\*For explanation, see Text p. 29.

†Note that each value listed in Column 4 is the average of several determinations from the same trace, one for each pair of maxima.

It may be well to point out that three corrections must be applied to the photometrically measured intensity ratio to obtain the true ratio of the mer-

cury isotopes. In the first place the intensity of the maxima decreases with decreasing order of interference. This is inherent in the etalon itself and was compensated for by a method suggested by Tolansky (15a). A smooth curve was drawn through the peaks of successive maxima for each separate component. The construction also averages deviations due to local fluctuations in plate graininess and sensitivity. The relative height of the peaks is given by the intersection of any vertical line with these curves.

Secondly, the intensity between orders of the  $\text{Hg}^{198}$  pattern contributes to the density of the  $\text{Hg}^{199}$  component. The converse does not produce a significant effect. To correct for this the lines were so chosen that the  $\text{Hg}^{199}$  maxima fell very nearly on the  $\text{Hg}^{198}$  minima. It is well known from interferometer theory (15b) that

$$[4a] \quad I_{\min} = I_{\max}/(1+F),$$

where  $I_{\min(\max)}$  = minimum (maximum) intensity in the interference pattern,

$$F = 4R/(1-R)^2,$$

$R$  = reflection coefficient.

For  $R = 0.805$  we have  $F = 84.75$ , and so

$$[4b] \quad I_{\min} = 0.012 I_{\max}.$$

From the measured height of the  $\text{Hg}^{198}$  peak the intensity  $I_{\min}$  of the  $\text{Hg}^{198}$  pattern was calculated from equation [4b]. This value was then subtracted from the measured  $\text{Hg}^{199}$  intensity.

Finally, to obtain the desired ratio, recall that the  $\text{Hg}^{199}$  maxima measured represent the contribution of only one component of the  $\text{Hg}^{199}$  structure. The intensity ratio among these various components may be calculated on the basis of the h.f.s. intensity formulae of E. L. Hill assuming strict  $J$ - $I$  coupling. Hence the measured  $\text{Hg}^{199}$  intensity must be increased by the appropriate factor, called  $k$  in Table I.

The value of the mercury ratio quoted above is subject to the accuracy of these corrections, in the second of which there is considerable uncertainty. The value is also strongly dependent on the precision with which the  $H$ - $D$  curve may be drawn. The difference in intensity between the  $\text{Hg}^{199}$  and  $\text{Hg}^{198}$  components was so great that the densities of both could not fall on the linear portion of the curve. The optimum condition appeared to be with the  $\text{Hg}^{198}$  maxima on the shoulder of the curve, the  $\text{Hg}^{199}$  maxima then approaching the linear portion.

Figs 2.(a), (b) are prints of the Fabry-Perot fringes and density calibrations of the lines  $\lambda\lambda 5461$  and  $4077 \text{ \AA}$ . Figs 3(a), (b), (c) reproduce the microphotometer tracings of  $\lambda\lambda 5461$ ,  $4077$ , and  $3125 \text{ \AA}$ .

The mercury was distilled from a gold sample irradiated with a neutron flux of  $4.6 \times 10^{13}$  neutrons per  $\text{cm}^2 \text{ sec}^*$ . The disintegration constant for  $\text{Au}^{198}$  is  $2.98 \times 10^{-6} \text{ sec}^{-1}$ . The cross section calculated from equation [3b] is then

$$\sigma = (1.78 \pm 0.10) \times 10^4 \text{ barns.}$$

\*We have recently been informed that the neutron flux may be in error by as much as 20%.

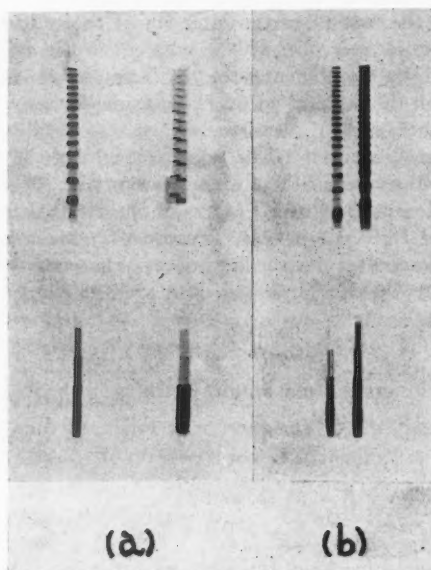


FIG. 2. (a) Hg  $\lambda 5461 \text{ \AA}$ , showing also yellow doublet  $\lambda 5770$  and  $\lambda 5790 \text{ \AA}$ .  
 (b) Hg  $\lambda 4077 \text{ \AA}$ , showing also the more intense line at  $\lambda 4046 \text{ \AA}$ .

This value is in good agreement with Hill's second estimate. Since the main difficulty in obtaining good accuracy for the capture cross section of  $\text{Au}^{198}$  arises in determining by densitometry the intensities of lines of widely different intensity, we intend to carefully repeat our measurements on favorable lines (small  $k$ ) with pure gold foil exposed to the highest neutron fluxes available.

#### CONCLUSIONS

The techniques of quantitative spectrochemical analysis can be applied, as in this work, to yield accurate values of collision cross sections. The  $\text{Hg}^{198}$  extracted from neutron-irradiated gold will contain traces of  $\text{Hg}^{199}$  in an amount proportional to the neutron flux density. Mercury produced by irradiating gold with high flux densities, say higher than  $10^{12}$  neutrons per  $\text{cm}^2 \text{ sec.}$ , is unsuited as a source for ultimate wavelength standards.

#### APPENDIX

##### (a) Absolute Wavelengths

On several of the Fabry-Perot plates the fringe diameters of 16  $\text{Hg}^{198}$  lines were measured on a Hilger comparator, and the fractional parts calculated by a least squares application (such as that outlined by Rolt and Barrell (13)). Using these values and Meggers' wavelengths (11) the exact order of interference for the 16 lines was determined by the method of exact fractions (4). The wavelengths of the lines were then recalculated relative to Meggers' value of

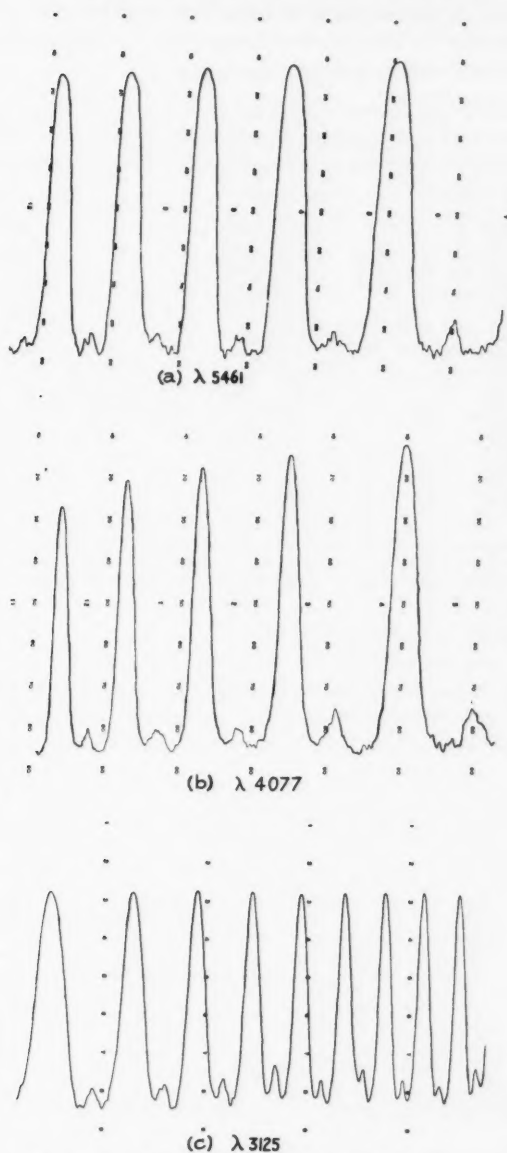


FIG. 3. (a) Microphotometer trace of Hg  $\lambda 5461$  Å.  
(b) Microphotometer trace of Hg  $\lambda 4077$  Å.  
(c) Microphotometer trace of Hg  $\lambda 3125$  Å.

3341.4814 Å. Our values are listed in Column 1, Table II, while for comparison the values obtained by Meggers and Kessler (11) and Burns and Adams (3) are reproduced in Columns 2 and 3 respectively.

TABLE II  
WAVELENGTHS OF Hg<sup>198</sup> (Å)

Bedford and Crooker	Meggers and Kessler (11)	Burns and Adams (3)
5790.6625	5790.6626	5790.6626
5769.5972	5769.5984	5769.5984
5460.7519	5460.7532	5460.7532
4358.3377	4358.3376	4358.3372
4077.8380	4077.8379	4077.8379
4046.5716	4046.5715	4046.5712
3663.2808	3663.2808	3663.2808
3654.8393	3654.8393	3654.8392
3650.1566	3650.1567	3650.1564
3341.4814	3341.4814	3341.4814
3131.8424	3131.8420	3131.8423
3131.5510	3131.5510	3131.5513
3125.6706	3125.6700	3125.6698
3021.4997	3021.4997	3021.4996
2967.2842	2967.2833	2967.2832
2536.5068	2536.5064	2536.5063

NOTE: The values in Columns 2 and 3 are relative to the green line,  $\lambda 5460.7532$  Å, while our values in Column 1 are relative to  $\lambda 3341.4814$  Å. The green line was not taken as standard in our calculations because only three fringes could be measured for it, giving reduced accuracy.

(b) The Hyperfine Structure Separations

For seven lines in which the Hg<sup>199</sup> structure was readily visible under the microscope the fringe diameters were measured on six different plates\* and the wave number separations calculated using Tolansky's rectangular array (15c). The results are summarized in Table III. The experimental values are shown

TABLE III  
HYPERFINE STRUCTURE OF Hg<sup>199</sup>

Wavelength (Å)	Wave number separations (cm. <sup>-1</sup> )		
	Bedford & Crooker	Brix & Kopferman (theoretical)	Schuler & Keyston
5461	-0.257	-0.259	†
4358	-0.212; +0.113	-0.222; +0.117	†
4077	-0.243	-0.243	-0.249
4046	+0.352	+0.363	+0.362
3341	-0.255	-0.235	
3125	+0.775	+0.775	
2536	+0.232	+0.221	+0.233

†Schuler and Keyston's values for these lines are given relative to Hg<sup>200</sup> and so are not quoted here.

in Column 2; theoretical values calculated from the data of Brix and Kopferman (2) are listed in Column 3; the values obtained by Schuler and Keyston

\*Only one measurement could be made for the line  $\lambda 3341$  Å.

(14) are reproduced in Column 4. In all cases the results quoted give the separation of the various  $\text{Hg}^{199}$  components from the single  $\text{Hg}^{198}$  component as zero.

#### ACKNOWLEDGMENTS

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## LETTERS TO THE EDITOR

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*Under this heading brief reports of important discoveries in physics may be published. These reports should not exceed 800 words and, for any issue, should be submitted not later than six weeks previous to the first day of the month of issue. No proof will be sent to the authors.*

### The Angular Distribution of the Gamma-rays from the Reaction $C^{12}(p, \gamma p')C^{12}$

In a recent experiment, Woodbury, Tollestrup, and Day (1) observed a gamma-ray which they attributed to the reaction  $C^{12}(p, \gamma p')C^{12}$ . The gamma-ray energy varied with bombarding energy according to the relation:

$$E_{\gamma} = 0.88(E_p - 0.45) \text{ Mev.},$$

and they concluded that the final state in the radiative process was the 2.369-Mev. first excited state of  $N^{14}$ , which in turn disintegrated into  $C^{12} + p$ . From their measurements at  $0^\circ$  and  $90^\circ$  they assumed that the angular distribution was nearly pure  $\sin^2 \theta$  and suggested that the capture mechanism was one of direct radiative capture.

We have measured the yield at five angles,  $0^\circ$ ,  $30^\circ$ ,  $45^\circ$ ,  $60^\circ$ , and  $90^\circ$ , using similar experimental procedure, and found that the angular distribution, after correction for geometry, can be represented by  $(0.02 \pm 0.02) + \sin^2 \theta$  at proton bombarding energies of 1.37 Mev. and 1.58 Mev. verifying the assumption of a  $\sin^2 \theta$  angular distribution and confirming the direct radiative capture mechanism.

We are indebted to Atomic Energy of Canada Ltd., for support during the course of this research.

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# CANADIAN JOURNAL OF PHYSICS

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